Quantum dissipation due to the interaction with chaotic degrees-of-freedom and the correspondence principle

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Both in atomic physics and in mesoscopic physics it is sometimes interesting to consider the energy time-dependence of a parametrically-driven chaotic system. We assume an Hamiltonian $\mathcal{H}(Q, P; x(t))$ where $x(t) = Vt$. The velocity V is slow in the classical sense but not necessarily in the quantum-mechanical sense. The crossover (in time) from ballistic to diffusive energy-spreading is studied. The associated irreversible growth of the average energy has the meaning of dissipation. It is found that a dimensionless velocity v_{PR} determines the nature of the dynamics, and controls the route towards quantal-classical correspondence (QCC). A perturbative regime and a non-perturbative semiclassical regime are distinguished.

Consider a system that is described by an Hamiltonian $\mathcal{H}(Q, P; x)$ where (Q, P) are canonical variables and x is a parameter. For example, x may represent the position of a large object ('piston' or 'Brownian particle') which is located inside a cavity, and the (Q, P) variables may describe the motion of one or few 'gas particles'. Assume that for $x(t){=}const$ the motion $(Q(t), P(t))$ is classically *chaotic*, and characterized by a correlation time τ_{cl}. Just for simplicity of the following presentation one may identify τ_{cl} with the ergodic time†. In this letter we are interested in the time-dependent case where $\dot{x}{=}V$. The velocity V is assumed to be slow in the classical sense. It is not necessarily slow in the quantum-mechanical (QM) sense. The notion of slowness is an important issue that we are going to discussed in detail. Because $V{\neq}0$, the energy is not a constant-of-motion. We study the crossover (in time) from ballistic to diffusive *energy-spreading*, and the associated irreversible growth of the average energy $\mathsf{E} = \langle \mathcal{H} \rangle$. By definition, this growth has the meaning of *dissipation*. It is common to *define* the dissipation coefficient μ via $\dot{\mathsf{E}} = \mu V^2$, where $\dot{\mathsf{E}}$ is the dissipation rate. The correspondence between quantal dissipation and classical dissipation should not be taken as obvious. It is expected that in the $\hbar \to 0$ limit the quantal μ will become similar to the classical μ. However, this is just an expectation. The actual 'proof' should be based on proper implementation of QM considerations. This considerations should further clarify what does it mean to have small $\hbar$, and what happens if $\hbar$ is not very small. The clarification of these issues is the main purpose of the present letter.

The interest in quantum-dissipation is very old [1–3]. There are few approaches to the subject. The most popular is the *effective-bath approach* [1]. When applied to 'our' problem (as defined above) it means that the chaotic (Q, P) degrees-of-freedom are replace by an effective-bath that has the same *spectral-properties*. This may be either harmonic-bath (with infinitely many oscillators) or random-matrix-theory (RMT) bath. It turns out that quantal-classical correspondence (QCC) is a natural consequence of this procedure: The dissipation coefficient μ turns out to be the same classically and quantum-mechanically. The effective-bath approach will not be adopted in this letter since its applicability is a matter of conjecture. We want to have a direct understanding of quantum-dissipation.

The understanding of *classical* dissipation, in the sense of this letter, is mainly based on [2,3]. Quantum-mechanically much less is known. Various *perturbative methods* have been used [3–5] in order to obtain an expression for the quantum-mechanical μ. These methods are (essentially) variations of the well known Fermi-Golden-Rule (FGR). The *simple* FGR expression for μ does not violate the expected correspondence with the classical result. However, this has been challenged most clearly by Wilkinson and Austin (W&A) [4]. They came up with a surprising conclusion that we would like to paraphrase as follows: A proper FGR picture, supplemented by an innocent-looking RMT assumption, leads to a *modified* FGR expression; In the $\hbar \to 0$ the modified FGR expression disagrees with the classical result. This observation was the original motivation for the present study.

The outline of this letter is as follows: In the next paragraph we are going to give a terse outline of our main observations. Then we start with a brief review of the classical picture. Most importantly: *It should be realized that the analysis of dissipation is reduced to the study of energy spreading.* This observation is valid classically as well as quantum-mechanically, and constitutes the corner-stone in the derivation of the universal fluctuation-dissipation (FD) relation. The rest of the paragraphs are dedicated to the presentation of the QM considerations. We are going to establish quantal-classical correspondence (QCC) in the limit $\hbar \to 0$ using semiclassical considerations. The detailed discussion of W&A's RMT approach is deferred to a long paper [6].

The main object to be discussed in this letter is the transition probability kernel $P_t(n|m)$. The variable m denotes the initial energy preparation of the system. It is a level index in the QM case. After time t the parameter $x{=}x(0)$ has a new value $x{=}x(t)$. Therefore it is possible to define a new set of (instantaneous) energy-eigenstates that are labeled by the index n. Thus, the kernel $P_t(n|m)$, regarded as a function of n, describes an

evolving energy-distribution. One may wonder whether the quantum-mechanical $P_t(n|m)$ is similar to the corresponding classical object. We shall distinguish between *detailed* QCC and *restricted* QCC. The latter term implies that only the second-moment of the spreading profile is being considered. The crossover from ballistic to diffusive energy-spreading happens at $t \sim \tau_{\rm cl}$. In order to *capture* this crossover within quantum-mechanics, a proper theory for the quantal $P_t(n|m)$ should be constructed. We shall define a scaled-velocity $v_{\rm PR}$. Our first main observation is that $v_{\rm PR} \ll 1$ is a necessary condition for the applicability of *perturbation theory*. In the perturbative regime the quantal $P_t(n|m)$ is *not* similar to the classical $P_t(n|m)$, there is no detailed QCC, but still one can establish *restricted* QCC. If $v_{\rm PR} \gg 1$, then the crossover at $t \sim \tau_{\rm cl}$ is out-of-reach for perturbation theory. Consequently non-perturbative approach is essential. This turns out to be the case in the limit $\hbar \to 0$. Our second main observation is that $v_{\rm PR} \gg 1$ is a necessary condition for *detailed* QCC. The latter is the consequence of *semiclassical* considerations.

The starting point for the classical theory of dissipation [2,4] is the statistical characterization of the fluctuating quantity $\mathcal{F}(t) = -(\partial\mathcal{H}/\partial x)$, assuming that x=*const.* Without loss of generality[‡] it is further assumed that the average force is $\langle\mathcal{F}\rangle_{\rm E} = 0$. The angular brackets stand for a microcanonical average over $(Q(0), P(0))$, where E is the energy. Note again that we still assume V=0. The temporal correlations of the stochastic-like force are $C_{\rm E}(\tau) = \langle\mathcal{F}(t)\mathcal{F}(t+\tau)\rangle$. It is assumed that the classical $C_{\rm E}(\tau)$ is characterized by a correlation time $\tau_{\rm cl}$. The intensity of fluctuations is described by the parameter $\nu = \tilde{C}(0)$. The power-spectrum of the fluctuations $\tilde{C}_{\rm E}(\omega)$ is defined via a Fourier transform.

For finite V the energy $\mathcal{E}(t) = \mathcal{H}(Q(t), P(t); x(t))$ is not a constant-of-motion. After time t the energy change is simply $(\mathcal{E}(t)-\mathcal{E}(0)) = -V\int_0^t \mathcal{F}(t)dt$. Let us assume that at t=0 we have a *microcanonical distribution* of initial 'points'. For short times $t \ll \tau_{\rm cl}$ one can prove that the evolving phase-space distribution is still confined to the initial energy surface. Thus the evolving distribution remains equal to the initial microcanonical distribution. This is the so-called classical *sudden approximation*. Squaring $(\mathcal{E}(t)-\mathcal{E}(0))$ and averaging over initial conditions we find that for short times we have a ballistic energy spreading:

$$\langle(\mathcal{E}(t) - \mathcal{E}(0))^2\rangle = C(0) \times (Vt)^2 \tag{1}$$

This ballistic behavior is just a manifestation of the parametric energy change $\delta\mathcal{E} = (\partial\mathcal{H}/\partial x)\delta x$. For longer times $(t \gg \tau_{\rm cl})$ we get a diffusive energy spreading:

$$\langle(\mathcal{E}(t) - \mathcal{E}(0))^2\rangle \approx 2D_{\rm E}t \tag{2}$$

$$D_{\rm E} = \tfrac{1}{2}\,\nu\, V^2 \tag{3}$$

Thus (for $t \gg \tau_{\rm cl}$) the evolving phase-space distribution is concentrated around the evolving energy surface $\mathcal{H}(Q, P; x(t)) = E$. This is the so-called classical *adiabatic approximation*. It becomes exact if one takes (after substitution of (3) into (2)) the formal limit $V \to 0$, keeping Vt constant. It should be evident that for finite V there is eventually a breakdown of the adiabatic approximation. The time $t_{\rm frc}$ of this breakdown is estimated in the next paragraph. The only approximation that was involved in the above analysis is that $\langle\mathcal{F}(t)\mathcal{F}(t+\tau)\rangle \approx C_{\rm E}(\tau)$. A strict equality applies (by definition) only if V=0. Detailed discussion of this approximation is quite straightforward [6]. It leads to the *classical slowness condition*. For the 'piston' example one easily concludes that the velocity V of the piston should be much smaller compared with the velocity of the gas particle(s).

The diffusion across the evolving energy surface leads to an associated systematic growth of the average energy E. This is due to the E-dependence of $D_{\rm E}$. The rate of energy growth is $\dot{\mathsf{E}} = \mu V^2$. The dissipation coefficient is related to the intensity of the fluctuations as follows:

$$\mu = \frac{1}{2}\frac{1}{g(E)}\frac{\partial}{\partial E}(g(E)\nu) \tag{4}$$

Here $g(E) = \partial_{\rm E}\Omega(E)$ is the classical density of states and $\Omega(E)$ is the phase-space volume which is enclosed by the respective energy surface. A canonical energy-averaging over the above FD relation leads to the familiar form $\mu = \nu/(2k_BT)$, where T is the temperature. The irreversible growth of the average energy ($\dot{\mathsf{E}}$=μV^2) implies that the fluctuating quantity $\mathcal{F}(t)$ has a non-zero average. Namely $\langle\mathcal{F}\rangle = -\mu V$. In case of the 'piston' example the latter is commonly named 'friction' force. The classical adiabatic approximation (2) is valid as long as the systematic growth of the average energy ($\dot{\mathsf{E}}t$) is much smaller than $\sqrt{2D_{\rm E}t}$. It leads to an estimate for the classical breaktime $t_{\rm frc} = \nu/(\mu V)^2$.

In order to make a smooth transition from the classical to the QM formulation we define the following kernels:

$$P_t(n|m) = \text{trace}(\ \rho_{n,x(t)}\ \mathcal{U}(t)\ \rho_{m,x(0)}\ ) \tag{5}$$

$$P(n|m) = \text{trace}(\ \rho_{n,x(t)}\ \rho_{m,x(0)}\ ) \tag{6}$$

In the classical context $\rho_{n,x}(Q, P)$ is defined as the microcanonical distribution that is supported by the energy-surface $\mathcal{H}(Q, P; x(t)) = E_n$. The energy E_n corresponds to the phase-space volume n=$\Omega(E)$. In the QM context $\rho_{n,x}(Q, P)$ is defined as the Wigner-function that represents the energy-eigenstate $|n(x)\rangle$. The phase-space propagator is denoted symbolically by $\mathcal{U}(t)$. In the classical case it simply re-positions points in phase-space. In the QM case it has a more complicated structure. The trace operation is just $dQdP$ integration. It is convenient to measure phase-space volume (n=$\Omega(E)$) in units of $(2\pi\hbar)^d$ where d is the number of degrees of freedom. This way we can obtain a 'classical approximation' for the QM kernel, simply by making n and m integer variables. If the 'classical approximation' is similar to the QM kernel, then we say that there is detailed

QCC[§]. If only the second-moment is similar, then we say that there is restricted QCC. The parametric kernel $P(n|m)$ is just the projection of the initial energy surface/eigenstate (labeled by m) on the new set of energy surfaces/eigenstates (labeled by n). For the parametric kernel, only the parametric change $\delta x \equiv Vt$ is important. The actual time (t) to realize this change is not important (by definition). This is not true for the actual kernel $P_t(n|m)$. The latter is defined as the projection of an evolving surface/eigenstate, where m is taken as the initial preparation. In the QM case we may use more conventional notation and write $P_t(n|m) = |\mathbf{U}_{nm}(t)|^2$ where $\mathbf{U}_{nm}(t) = \langle n(x(t))|\mathbf{U}(t)|m(0)\rangle$ are the matrix elements of the evolution operator. Similarly, the parametric kernel $P(n|m)$ is related to the transformation matrix $\mathbf{T}_{nm}(x) = \langle n(x)|m(0)\rangle$.

Let us paraphrase the classical discussion that leads to (4). By definition, the departure of $P_t(n|m)$ from $P(n|m)$ marks the breakdown of the sudden approximation. The spreading of $P_t(n|m)$, which is implied by (2), reflects the deviation from the (classical) adiabatic approximation. The stochastic nature of the spreading on long times implies a systematic growth of the average energy. The considerations that lead to the FD relation (4) are of quite general nature, provided *genuine* diffusive behavior is established: It is argued [6] that $P_t(n|m)$ can be written as the convolution of N kernels $P_{t_1}(n|m)$ such that $t = Nt_1$ and $\tau_{\text{cl}} \ll t_1 \ll t_{\text{frc}}$. Using general argumentation, the same as in the derivation of the 'central limit theorem', one concludes that $P_t(n|m)$ obeys a diffusion equation. The diffusion coefficient D_{E} is determined by the second-moment of $P_{t_1}(n|m)$, and hence it is given by (3). Most importantly, higher-moments of $P_{t_1}(n|m)$ become irrelevant. The validity of the above argumentation is conditioned by the requirement of having the separation of time scales $\tau_{\text{cl}} \ll t_{\text{frc}}$. This is a main ingredient in the *classical definition of slowness*. In the QM case we can try to use the same reasoning in order to derive a FD relation that corresponds to (4). The crucial step is to establish the diffusive behavior (2) for a limited time scale which is required to be much longer than τ_{cl}. The quantum-mechanical D_{E} will hopefully corresponds to (3). This correspondence is not a-priori guaranteed. This is the main issue that we are going to address in the rest of this letter. First we discuss the conditions for having QCC for the parametric kernel $P(n|m)$. Then we discuss the departure of $P_t(n|m)$ from $P(n|m)$. The conditions for having either detailed or at least restricted QCC will be specified.

The quantal $\rho_{n,x}(Q,P)$, unlike its classical version, has a non-trivial transverse structure. For a curved energy surface the transverse profile looks like Airy function and it is characterized by a width $\Delta_{\text{SC}} = (\varepsilon_{\text{cl}}(\hbar/\tau_{\text{cl}})^2)^{1/3}$ where ε_{cl} is a classical energy scale. For the 'piston' example $\varepsilon_{\text{cl}}{=}E$ is the kinetic energy of the gas particle. Given a parametric change $\delta x \equiv Vt$ we can define a classical energy scale $\delta E_{\text{cl}} \propto \delta x$ via (1). This parametric energy scale characterizes the transverse distance between the intersecting energy-surfaces that support $|m(x)\rangle$ and $|n(x{+}\delta x)\rangle$. Considering $P(n|m)$, it should be legitimate to neglect the transverse profile of Wigner function provided $\delta E_{\text{cl}} \gg \Delta_{\text{SC}}$. This condition can be cast into the form $\delta x \gg \delta x_{\text{SC}}$ where $\delta x_{\text{SC}} = \Delta_{\text{SC}}/(\sqrt{\nu^{\text{cl}}/\tau_{\text{cl}}})$. If $\delta x \ll \delta x_{\text{SC}}$ we cannot argue that there is detailed QCC. On the other hand we can not rule out such QCC. We shall come back to this issue shortly.

If δx is sufficiently small it should be possible to get an approximation for $\mathbf{T}_{nm}(x)$, and hence for $P(n|m)$, via perturbation theory. It turns out that in the perturbative regime $P(n|m)$ is characterized by a core-tail structure that does not correspond to the classical $P(n|m)$. Detailed definition and discussion of the core-tail structure [6] are not required for the following considerations. The only important (non-trivial) observation is that in-spite of the lack of detailed QCC there is still restricted QCC. Namely, the second moment is still given by δE_{cl}. An estimate for the breakdown of perturbation theory can be easily obtained using heuristic considerations as follows: The range of first-order transitions is determined by the bandwidth $\Delta_b = \hbar/\tau_{\text{cl}}$ of the matrix $(\partial\mathcal{H}/\partial x)_{nm}$. See [5]. In the perturbative regime $P(n|m)$ is non-vanishing only if $|E_n - E_m| < \Delta_b$. Perturbation theory is inapplicable unless the spreading is on an energy-scale $\delta E_{\text{cl}} \ll \Delta_b$. This condition can be cast into the form $\delta x \ll \delta x_{\text{prt}}$ where

$$\delta x_{\text{prt}} \;=\; \hbar \;/\; \sqrt{\nu^{\text{cl}}\tau_{\text{cl}}} \qquad (7)$$

We have $\delta x_{\text{prt}} \propto \hbar$ and $\delta x_{\text{SC}} \propto \hbar^{2/3}$. Therefore, typically, the two parametric scales are well separated: We do not have a theory for the intermediate parametric regime $\delta x_{\text{prt}} \ll \delta x \ll \delta x_{\text{SC}}$. The only thing that we can say with confidence is that the core-tail structure of $P(n|m)$ is washed away once $\delta x > \delta x_{\text{prt}}$. We do not know whether there is an additional crossover once we go paste δx_{SC}. Therefore it is more meaningful to state that $\delta x > \delta x_{\text{prt}}$ is a *necessary* condition for detailed QCC, rather than specifying the *sufficient* condition $\delta x > \delta x_{\text{SC}}$.

We turn now to discuss the actual transition probability kernel $P_t(n|m)$. Recall that our objective is to *capture* the crossover at $t \sim \tau_{\text{cl}}$. Therefore it is essential to distinguish between two possibilities: If $V\tau_{\text{cl}} \ll \delta x_{\text{prt}}$ it means that the crossover happens in a regime where perturbation theory is still valid. On the other hand if $V\tau_{\text{cl}} \gg \delta x_{\text{prt}}$ it means that the crossover is out-of-reach for perturbation theory. It is then essential to use non-perturbative considerations. The sufficient condition for the applicability of semiclassical theory is $V\tau_{\text{cl}} \gg \delta x_{\text{SC}}$. Thus we come to the conclusion that the following generic dimensionless parameter controls QCC:

$$v_{\text{PR}} \;=\; \text{scaled velocity} \;=\; \sqrt{D_{\text{E}}\tau_{\text{cl}}}/\Delta_b \qquad (8)$$

If $v_{\text{PR}} \ll 1$ then it is feasible to extend perturbation theory beyond τ_{cl}. This issue is discussed in the next paragraphs. In the limit $\hbar \to 0$ we have $v_{\text{PR}} \gg 1$ and per-

turbation theory is inapplicable. However, if $v_{\rm PR}$ is sufficiently large then semiclassical consideration can be used in order to argue that the classical result is valid also in the QM domain. Before we go on, we should mention an additional restriction on QCC, that pertains to $P_t(n|m)$. The evolving (classical) distribution $\mathcal{U}(t)\rho_{m,x(0)}$ becomes more and more convoluted as a function of time. This is because of the mixing behavior that characterizes chaotic dynamics. For $t > t_{\rm scl}$ the intersections with a given instantaneous energy surface n become very dense, and no-longer can be resolved by quantum-mechanics. The semiclassical breaktime $t_{\rm scl}$ is related to the failure of the stationary phase approximation [7]. In order to establish the crossover from ballistic to diffusive energy spreading using a semiclassical theory we should satisfy the condition $\tau_{\rm cl} < t_{\rm scl}$. This velocity-independent condition turns out to be not very restrictive [7], and we can safely assume that it is typically satisfied.

We are now left with the question whether restricted QCC is maintained in the perturbative regime for $t > \tau_{\rm cl}$. Indeed, if $v_{\rm PR} \ll 1$, perturbation theory can be used in order to get an approximation for $\mathbf{U}_{nm}(t)$ and hence for the respective kernel $P_t(n|m)$. Then it is possible to calculate the second-moment of the energy-spreading and to obtain an expression that looks like (2). Expression (3) for $D_{\rm E}$ applies, provided ν is replaced by an effective noise intensity:

$$\nu^{\rm eff} \ = \ \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \tilde{C}_{\rm E}(\omega)\tilde{F}(\omega) \qquad (9)$$

The detailed derivation of this result will be presented elsewhere [6]. Formally it looks exactly the same as either the simple FGR result [3] or W&A's result [4]. But this formal similarity is quite misleading. The difficult issue is how $\tilde{F}(\omega)$ is defined. This function describes the *effective* power spectrum of the driving force. It is the Fourier transform of a correlation function $F(\tau)$ with the convention $F(0)$=1. The latter is characterized by a correlation time τ_c. The demonstration that the intrinsic τ_c is much larger than $\tau_{\rm cl}$ is the main challenge of the theory [6]. The implied *restricted* QCC is explained below.

We turn to discuss the physical consequences of (9). For this purpose we should first explain how the QM fluctuation-spectrum look like. The fluctuating quantity $\mathcal{F}(t)$ should be handled as an operator. The quantal $C_{\rm E}(\tau)$ is similar to the classical $C_{\rm E}(\tau)$ provided $\tau \ll t_{\rm H}$. See [5]. Here $t_{\rm H} = \hbar/\Delta$ is the Heisenberg time and Δ is the mean level spacing. The associated $\tilde{C}_{\rm E}(\omega)$ reflects the discrete nature of the energy levels, but its envelope is classical-like. The *conditions for restricted* QCC are obtained by inspection of (9): The function $C_{\rm E}(\tau)$ is characterized by two distinguished time scales, which are $\tau_{\rm cl}$ and $t_{\rm H}$. The function $F(\tau)$ is characterized by a single time scale τ_c. Accordingly we have the following possibilities: If $\tau_c \ll \tau_{\rm cl}$ then the transitions are *band-limited* and we get $\nu^{\rm eff} \approx (\tau_c/\tau_{\rm cl}) \times \nu^{\rm cl}$. If $\tau_c \gg \tau_{\rm cl}$ then the transitions are *resonance-limited* and we get $\nu^{\rm eff} \approx \nu^{\rm cl}$. In the latter case $\tilde{F}(\omega)$ is essentially like a delta function. However $\tilde{F}(\omega)$ should not be too narrow. Namely, if $\tau_c \gg t_{\rm H}$ then the effective noise intensity becomes vanishingly small. This is because individual levels are resolved. The condition $\tau_c \gg t_{\rm H}$ is satisfied only for extremely-slow velocities. This is the so-called QM-adiabatic regime. There Landau-Zener transitions are the ultimate mechanism for energy-spreading and dissipation [4], and QCC is not a-priori guaranteed.

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† By definition the correlation time $\tau_{\rm cl}$ pertains to $\mathcal{F}(t)$ which is defined later. In specific examples it may be smaller then the ergodic time. This is the case with the 'piston' example. Assuming that successive collisions with its faces are uncorrelated, $\tau_{\rm cl}$ is just the duration of a collision. (For hard walls $\tau_{\rm cl} \sim 0$). On the other hand the ergodic time is related to the ballistic time.

‡ Without loss of generality we assume $\langle \mathcal{F} \rangle = 0$. This is equivalent to having phase-space volume $\Omega(E,x)$ which is independent of x. (Such is the case for a gas particle which is affected by collisions with a small rigid body that is translated inside a large cavity). A non-zero average component of $(\partial\mathcal{H}/\partial x)$ corresponds to a conservative force, and should be subtracted from the definition of $\mathcal{F}$.

§ 'Detailed QCC' does not mean complete similarity. The classical kernel is typically characterized by various non-Gaussian features, such as sharp cutoffs, delta-singularities and cusps. These features are expected to be smeared in the QM case. The discussion of the latter issue is beyond the scope of the present letter.

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